Endogenous Market-Clearing Prices and Reference Point Adjustment

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Social Computing, Behavioral Modeling and Prediction
1. Context


- Discovered preference hypothesis (Plott 1996): preferences converge to the same underlying preferences regardless of the market mechanism.

- Experimentalists want individuals to reveal their preferences truthfully: they use incentive-compatibility constraints.
2. Auction mechanisms and IC

- The BDM and NPA mechanisms are often used in experiments.

- **BDM**: price is exogenously and randomly drawn from a price list. If the agent bids above the price, she receives the good and pays the drawn price.

- **NPA**: price is endogenously determined. The clearing is the following:

  1. Each bidder submits a bid;
  2. All bids are rank-ordered;
  3. Number between 2 and \( n \) (\( n \) the number of bidders) is randomly selected;
  4. Item is sold to each of the \( (n-1) \) highest bidders at the selected price.
2.1. Asocial IC

- The BDM and NPA are theoretically proved to be incentive-compatible.

- Expected utility payoff corresponds to:

\[
E(u_i) = \int_p^{b_i} u_i(v_i - p) f_i(p) dp + \int_{b_i}^{\bar{p}} u_i(0) dp
\]

- The maximum over \( b_i \) occurs when:

\[
\frac{\partial E(u_i)}{\partial b_i} = u_i(v_i - b_i) f_i(b_i) = 0
\]

- The agent maximizes her expected utility when she bids her true value.
2.2. Computational constraint

- IC requires that truth telling is best averaged over the space of types and prior beliefs of all other agents.

- When the agents’ types are unknown, each agent must know the distribution of types of all other agents. Is this plausible in reality?

- In absence of common knowledge over types’ space and prior beliefs, agents do not compute the equilibrium (Saran and Serrano 2007).

- It is realistic to stress that agents observe how others value the good, and some kind of equilibrium emerges (Boutilier *et al.* 2000).
2.3. Independent private values’ constraint

- IC works with independent private values: agent’s value is independently drawn from a commonly known distribution.

- But, valuation depends on agent’s and others’ information: affiliated private values phenomenon (Milgrom and Weber 1982, see also Klemperer 1999).

- A higher value for one agent makes higher values for other agents more likely (Kagel 1995).

- Posted prices have a statistically significant impact on bids submitted in subsequent rounds (Corrigan and Rousu 2006).

- When prices are issued from the bids, are we still verifying IC?
2.4. Social IC

- Following Myerson (1983), suppose a correlated equilibrium.

- Public signaling from a trusted third party recommends a strategy that generates higher expected utility payoffs.

\[
\begin{array}{c|cc}
\text{Agent 1} & \text{fixes} & \text{adjusts} \\
\hline
\text{fixes} & v_1^f, v_2^f & v_1^f, v_2^a \\
\text{adjusts} & v_1^a, v_2^f & 0, 0 \\
\end{array}
\]

- Agents learn their recommended strategy from posted prices: \( \langle v_1^f \rangle \) fixing the true value and \( \langle v_1^a \rangle \) adjusting the true value after agents discovered it.
- Recommendations are issued from $f_\rho$ in $\Delta(\Phi)$. The expected utility payoff under a correlated strategy $f_\rho$ is $u_i(f_\rho)$.

- There is an IC equilibrium if and only if

$$u_i(f_\rho \mid t_i) \geq \sum_{t \in T} \sum_{\phi \in \Phi} f_\rho(\phi') u_i(\phi_{-i}, \delta_i(\phi) \mid t_i) :=$$

$$v_if_p(b^\circ_{-i}, b^\circ_i) - p_i(b^\circ_{-i}, b^\circ_i) \geq v_if_p(b^\circ_{-i}, b'_i) - p_i(b^\circ_{-i}, b'_i),$$

- An additional probability constraint is added

$$\sum_{\phi' \in \Phi} f_\rho(\phi \mid t) = 1, \quad f_\rho(\phi' \mid t) \geq 0$$

- Thus the correlated equilibrium that maximizes expected utility payoff is

$$f_\rho(v^f_1, v^f_2) = f_\rho(v^a_1, v^f_2) = f_\rho(v^f_1, v^a_2) = 1/3, \quad f_\rho(v^a_1, v^a_2) = 0$$
PROPOSITION 1.

When true-type agents follow public signals recommended from endogenous market-clearing prices, they operate in a correlated equilibrium.
3. The behavioral model

- Model based upon the anchoring-and-adjustment heuristic.

- Reference point adjustment analogous to rank-dependent expected utility.

- The anchor is updated after observing the public signal.

- In order to increase her expected utility payoff, depending on whether \( p_k > r_{k-1} \) or \( p_k < r_{k-1} \) at the \( k \)th round, the agent scales her bid up or down.

- Weighted memory register: replies from posted prices memory weighting.
3.1. Weighted memory register

- Probability weighting (Tversky and Wakker 1995). In parallel, we have weighted memory register with \( w(0) = 0 \) and \( w(1) = 1 \).

- Bidders accumulate ranks and weigh their cumulative ranks

\[
(w(\theta_n),...,w(\theta_1)) := (w(1/n),...,w(n/n))
\]

- **Unbounded memory weighting** is an equal weighting between two rounds.

\[
w(\theta_k) - w(\theta_{k-1}) := w(k/n) - w((k-1)/n)
\]

- **Bounded memory weighting** overweighs the anchor (guarantee for truthful bidding) and the last posted price and underweighs posted prices in-between.
Bounded memory weighting

\[ w(1) - w(1 - (1/n)) \]

\[ w(1 - (1/n)) - w(1/n) \]

\[ w(1/n) - w(0) \]
**PROPOSITION 2.**
An agent is truth-telling inasmuch as she behaves as a (boundedly rational) utility payoff maximizer, i.e. so long as she plays pursuant to the bounded memory weighting.
3.2. The empirical study

- Test of the model through the BDM and NPA experiments on carbon offsetting (Dragicevic and Ettinger 2010).

- The $t$-test fails to reject the null hypothesis that the theoretical bonds in offers and the real bonds in offers come from the same distribution at the $p < 0.05$ level for both BDM and NPA.

- We do not reject the null hypothesis with bids from NPA either.

- We reject the null hypothesis with BDM bids.

- SSE residuals significantly lower with bounded memory weighting than with unbounded memory weighting: bidders and offerers are truthful!
<table>
<thead>
<tr>
<th>$\gamma$ estimate</th>
<th>Unbounded memory</th>
<th>Bounded memory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BDM</td>
<td>NPA</td>
</tr>
<tr>
<td>Bidders</td>
<td>1.18 (0.02)</td>
<td>1.24 (0.03)</td>
</tr>
<tr>
<td>Offerers</td>
<td>1.19 (0.02)</td>
<td>1.17 (0.03)</td>
</tr>
</tbody>
</table>

![Graph](image-url)
### Expected and real gainers from adjustment

<table>
<thead>
<tr>
<th>per cent</th>
<th>BDM WTP</th>
<th>BDM WTA</th>
<th>NPA WTP</th>
<th>NPA WTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gainers</td>
<td>2.63</td>
<td>4.17</td>
<td>10.00</td>
<td>10.94</td>
</tr>
<tr>
<td>Real gainers</td>
<td>0.00</td>
<td>2.78</td>
<td>3.33</td>
<td>3.13</td>
</tr>
</tbody>
</table>

### Expected and real gains from adjustment

<table>
<thead>
<tr>
<th>on average</th>
<th>BDM WTP</th>
<th>BDM WTA</th>
<th>NPA WTP</th>
<th>NPA WTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gain</td>
<td>0.13</td>
<td>–0.36</td>
<td>0.72</td>
<td>0.09</td>
</tr>
<tr>
<td>Real gain</td>
<td>0.13</td>
<td>–0.22</td>
<td>0.26</td>
<td>0.08</td>
</tr>
</tbody>
</table>
4. Conclusions

- We show that agents bid according to the anchor-and-adjustment heuristic using public signals encoded in a memory weighting register.

- The empirical test of the model confirms the capacity of both the BDM and NPA auction mechanisms to be demand-revealing.

- IC need not be excluded in presence of reference point adjustment, as long as one verifies heavy weighting of the anchor.

- We thus suggest a form of social rationality where the correlated equilibrium plays a key role.